$$U'_{r1} = R'_r \sum_s d_s q_s^{(3)} + S'_r \sum_\eta d_\eta q_\eta^{(4)} + P'_r. \tag{16}$$

The remainder of the procedure is as above for consistent grids, except that, in calculating the temperature in the subregion, the heat fluxes at the rod boundaries are determined by interpolation with respect to their values at the node points.

Note that this method may be extended without difficulty to systems containing three-dimensional subregions.

NOTATION

T, U, temperature; θ, temperature at the boundary between subregions; q, heat flux; t, time; x_α, coordinate; β, dimensionality of subregion; cρ, volume specific heat; λ, thermal conductivity.

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METHOD OF DETERMINING THE THERMOPHYSICAL PROPERTIES OF ORTHOTROPIC MATERIALS FROM THE SOLUTION OF A TWO-DIMENSIONAL INVERSE HEAT-CONDUCTION PROBLEM

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An unsteady two-dimensional inverse coefficient problem of heat conduction is formulated mathematically and solved.

Recent years have seen the active development of methods of determining the thermophysical properties (TPC) of materials which make use of empirical data obtained from the unsteady heating of specimens [1-3]. The theoretical foundation of these methods are mathematical formulations of unsteady inverse coefficient problems of heat transfer, which are usually inverse heat-conduction problems (ICP). The overwhelming majority of ICP mathematical formulations are based on the assumption that heat transfer is unidimensional – an assumption which is keeping investigators from making thermophysical studies more informative and applicable to a broader range of temperatures. This is particularly true in regard to comprehensive study of the TPC of anisotropic materials, the use of concentrated energy flows for heating materials, and study of TPC directly on objects of complex structure and shape. The practical resolution of these issues – which will mark a new step in the methodology of thermophysical studies – should begin with the mathematical formulation and solution of unsteady multidimensional ICP.

We will examine the mathematical formulation and solution algorithm of a two-dimensional nonlinear coefficient ICP. Let the object in thermal tests be a flat rectangular specimen made of a homogeneous orthotropic material in which the principal axes of the thermal conductivity tensor coincide with the coordinate axes x_1 and x_2. The TPC of the material – the volumetric heat capacity cρ and the thermal conductivities λ_{x1}, λ_{x2} – are dependent on temperature. The initial temperature of the specimen and the heat-transfer conditions on its faces are known. During heating (cooling), temperature is measured at several points of the

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specimen. It is assumed that the temperature dependence of the volumetric heat capacity $c\rho(T)$ is known. We need to determine the temperature dependences of thermal conductivity $\lambda_{x_1}(t)$, $\lambda_{x_2}(T)$.

The above physical model of heat transfer corresponds to the following system of equations:

$$c\rho(T)\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x_1}\left(\lambda_{x_1}(T)\frac{\partial T}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\lambda_{x_2}(T)\frac{\partial T}{\partial x_2}\right); \tag{1}$$

$$\tau=0 \quad T=T_0; \tag{2}$$

$$x_1=0 \quad -r_{11}\lambda_{x_1}(T)\frac{\partial T}{\partial x_1}=(r_{11}-1)T+\varphi_{11}(T,\ \tau,\ x_2); \tag{3}$$

$$x_1=l \quad r_{12}\lambda_{x_1}(T)\frac{\partial T}{\partial x_1}=(r_{12}-1)T+\varphi_{12}(T,\ \tau,\ x_2); \tag{4}$$

$$x_2=0 \quad -r_{21}\lambda_{x_2}(T)\frac{\partial T}{\partial x_2}=(r_{21}-1)T+\varphi_{21}(T,\ \tau,\ x_1); \tag{5}$$

$$x_2=h \quad r_{22}\lambda_{x_2}(T)\frac{\partial T}{\partial x_2}=(r_{12}-1)T+\varphi_{22}(T,\ \tau,\ x_1); \tag{6}$$

$$r=0 \text{ or } 1.$$

With boundary conditions of the first type, $\varphi(T,\ \tau,\ x) = T_w(\tau,\ x)$. When the boundary conditions of the second and third types are used, $\varphi(T,\ \tau,\ x) = A(T_w)q_w(\tau,\ x) + \varepsilon(T_w)\sigma T_w{}^4(\tau,\ x) - \alpha_f(T_w(\tau,\ x) - T_f)$.

The inverse problem of determining $\lambda_{x_1}(T)$ and $\lambda_{x_2}(T)$ from experimental data on the thermal state of the specimen and assigned heat-transfer conditions will be solved in an extremal formulation [4]. We construct a computing algorithm for determining $\lambda_{x_1}(T)$ and $\lambda_{x_2}(T)$ that ensures the minimum of the functional

$$S=\int_0^{\tau_e}\sum_{m=1}^{M}(T(m,\ \tau)-T_e(m,\ \tau))^2\,d\tau. \tag{7}$$

Equations (1)-(6), together with (7), constitute the mathematical formulation of the ICP.

We represent the sought relations $\lambda_{x_1}(T)$ and $\lambda_{x_2}(T)$ in parametric form:

$$\lambda_{x_\alpha}(T)=\{\lambda_{\alpha k},\quad k=\overline{1,\ K}\},\quad \alpha=1,\ 2, \tag{8}$$

where $\lambda_{\alpha k}$ are values of thermal conductivity corresponding to the temperature T_k, $k = \overline{1,\ K}$. Thermal conductivity changes according to a linear law between nodal values. The number of sought parameters should not exceed the number of independent temperature measurements. The number of parameters K is usually designated in accordance with the presumed form of the temperature dependence of the TPC.

As a result of representation (8), we have a problem of parametric identification in a space of 2K parameters. The use of the methods of iterative regularization is effective in solving ill-conditioned problems [4]. In this case, the process of minimizing functional (7) consists of a series of successive iterations. In each iteration, a gradient method [5] is used to determine the search direction, and the minimum is found in the given direction. One criterion for terminating the iteration is the condition $S \le \delta^2$, where $\delta^2=\int_0^{\tau_e}\sum_{m=1}^{M}\delta T^2 d\tau$, δT is the temperature measurement error.

We numerically solve problem (1)-(6) to calculate the functional (7) [determine the quantity $T(m,\ \tau)$ in the functional]. Here, we introduce a difference grid:

a space grid on the interval $[0,\ \ell]$:

$$\bar{\omega}_l=\{x_1=(i-1)\Delta x_1,\quad \Delta x_1>0,\quad i=\overline{1,\ N_1};\quad \Delta x_1(N_1-1)=l\};$$

a space grid on the interval [0, h]:

$$\bar{\omega}_h = \{x_2 = (j-1)\,\Delta x_2,\quad \Delta x_2 > 0,\quad j = \overline{1, N_2};\quad \Delta x_2 (N_2 - 1) = h\};$$

and a time grid on the interval $[0, \tau_e]$:

$$\bar{\omega}_\tau = \{\tau^n = n\Delta\tau,\ \Delta\tau > 0,\ n = \overline{0, N_3};\quad \Delta\tau N_3 = \tau_e\}.$$

We designate $T^n = T(x_\alpha, \tau^n)$; $c^n = c(x_\alpha, \tau^n)$; $\lambda^n = \lambda(x_\alpha, \tau^n)$, $\alpha = 1, 2$. Using a locally unidimensional scheme [6], we obtain the following for the first time half-step:

$$-r_{11,j}\,(a_{1,j}^{n+1}\,(T_{2,j}^{n+\frac{1}{2}} - T_{1,j}^{n+\frac{1}{2}}) - c\rho_{1,j}^{n+1}\,(T_{1,j}^{n+\frac{1}{2}} - T_{1,j}^{n})/2\Delta\tau) = ((r_{11,j} - 1)\,T_{1,j}^{n+\frac{1}{2}} + \varphi_{11}^{n+1})/\Delta x_1; \tag{9}$$

$$c\rho_{i,j}^{n+1}\,(T_{i,j}^{n+\frac{1}{2}} - T_{i,j}^{n})/\Delta\tau = a_{i,j}^{n+1}\,(T_{i+1,j}^{n+\frac{1}{2}} - T_{i,j}^{n+\frac{1}{2}}) - a_{i-1,j}^{n+1}\,(T_{i,j}^{n+\frac{1}{2}} - T_{i-1,j}^{n+\frac{1}{2}}); \tag{10}$$

$$r_{12,j}\,(a_{N_1-1,j}^{n+1}\,(T_{N_1,j}^{n+\frac{1}{2}} - T_{N_1-1,j}^{n+\frac{1}{2}}) - c\rho_{N_1,j}^{n+1}\,(T_{N_1,j}^{n+\frac{1}{2}} - T_{N_1,j}^{n})/2\Delta\tau) = ((r_{12,j} - 1)\,T_{N_1,j}^{n+\frac{1}{2}} + \varphi_{12,j}^{n+1})/\Delta x_1, \tag{11}$$

where

$$a_{i,j}^{n+1} = \frac{1}{\Delta x_1^2}\,\frac{2\lambda_{x_1,i,j}^{n+1}\cdot\lambda_{x_1,i+1,j}^{n+1}}{(\lambda_{x_1,i,j}^{n+1} + \lambda_{x_1,i+1,j}^{n+1})}.$$

For the second time half-step $i = \overline{1, N_1}$:

$$-r_{21,i}\,(b_{i,1}^{n+1}\,(T_{i,2}^{n+1} - T_{i,1}^{n+1}) - c\rho_{i,1}^{n+1}\,(T_{i,1}^{n+1} - T_{i,1}^{n+\frac{1}{2}})/2\Delta\tau) = ((r_{21,i} - 1)\,T_{i,1}^{n+1} + \varphi_{21,i}^{n+1})/\Delta x_2; \tag{12}$$

$$c\rho_{i,j}^{n+1}\,(T_{i,j}^{n+1} - T_{i,j}^{n+\frac{1}{2}})/\Delta\tau = b_{i,j}^{n+1}\,(T_{i,j+1}^{n+1} - T_{i,j}^{n+1}) - b_{i,j-1}^{n+1}\,(T_{i,j}^{n+1} - T_{i,j-1}^{n+1}); \tag{13}$$

$$r_{22,i}\,(b_{i,N_2-1}^{n+1}\,(T_{i,N_2}^{n+1} - T_{i,N_2-1}^{n+1}) - c\rho_{i,N_2}^{n+1}\,(T_{i,N_2}^{n+1} - T_{i,N_2}^{n+\frac{1}{2}})/\Delta\tau) = ((r_{22,i} - 1)\,T_{i,N_2}^{n+1} + \varphi_{22,i}^{n+1})/\Delta x_2, \tag{14}$$

where

$$b_{i,j}^{n+1} = \frac{1}{\Delta x_2^2}\,\frac{2\lambda_{x_2,i,j}^{n+1}\,\lambda_{x_2,i,j+1}^{n+1}}{\lambda_{x_2,i,j}^{n+1} + \lambda_{x_2,i,j+1}^{n+1}}.$$

System (9)-(14) is solved in each half-step by the trial run method.

We use the Davidson–Fletcher–Powell method [5] to determine the search direction for each new iteration. The values of the gradients of the functional $\partial S/\partial\lambda_{1k}$, $\partial S/\partial\lambda_{2k}$, $k = \overline{1, K}$ needed for this method are calculated from the solution of the problem conjugate with (9)-(14) constructed on the basis of the recommendations in [1, 7]. The solution of the conjugate problem in regard to specific specimen heat-transfer conditions is one of the most important and most complex elements of the algorithm for the solution of the inverse heat-conduction problem. In our case, the calculation of 2K gradients in each iteration requires only one solution of the conjugate problem. The computing time necessary here is comparable to the time needed to solve direct heat-conduction problem (9)-(24).

The solution of the conjugate problem of (9)-(14) is constructed as follows. The initial value of the grid function of the conjugate equation is assigned with $\tau_e = \Delta\tau N_3$: $\gamma_{i,j}^{N_3+1} = 0$, $i = \overline{1, N_1}$; $j = \overline{1, N_2}$. Calculations are performed in the "reverse" direction from $\tau = \tau_e$ to $\tau = 0$. For the first time half-step, the calculations are performed along the axis $j = \overline{1, N_2}$:

$$\gamma_{1,j}^{n-\frac{1}{2}}\,(r_{11}\,(2a_{1,j}^{n} + c\rho_{1,j}^{n}/\Delta\tau) + (r_{11,j} - 1)/\Delta x_1) = r_{11,j}\,(\gamma_{2,j}^{n-\frac{1}{2}}\,a_{1,j}^{n} + \gamma_{1,j}^{n}\,c\rho_{1,j}^{n}/\Delta\tau) + \partial S/\partial T_{1,j}; \tag{15}$$

$$2\gamma_{1,j}^{n-\frac{1}{2}}\,r_{11,j}\,a_{1,j}^{n} - \gamma_{2,j}^{n-\frac{1}{2}}\,(a_{1,j}^{n} + a_{2,j}^{n} + c\rho_{2,j}^{n}/\Delta\tau) + \gamma_{3,j}^{n-\frac{1}{2}}\,a_{2,j}^{n} = -\gamma_{2,j}^{n}\,c\rho_{2,j}/\Delta\tau - \partial S/\partial T_{2,j}; \tag{16}$$

$$\gamma_{i-1,j}^{n-\frac{1}{2}} a_{i-1,j}^n - \gamma_{i,j}^{n-\frac{1}{2}} (a_{i,j}^n + a_{i-1,j}^n + c\rho_{i,j}^n/\Delta\tau) + + \gamma_{i+1,j}^{n-\frac{1}{2}} a_{i,j}^n = -\gamma_{i,j}^n c\rho_{i,j}^n/\Delta\tau - \partial S/\partial T_{i,j}, \quad i = \overline{3,\ N_1-2}; \tag{17}$$

$$\gamma_{N_1-2,j}^{n-\frac{1}{2}} a_{N_1-2,j}^n - \gamma_{N_1-1,j}^{n-\frac{1}{2}} (a_{N_1-1,j}^n + a_{N_1-2,j}^n + c\rho_{N_1-1,j}/\Delta\tau) + + 2\gamma_{N_1,j}^{n-\frac{1}{2}} r_{12,j} a_{N_1-1,j}^n = -\gamma_{N_1-1,j}^n c\rho_{N_1-1,j}^n/\Delta\tau - \partial S/\partial T_{N_1-1,j}; \tag{18}$$

$$\gamma_{N_1,j}^{n-\frac{1}{2}} (r_{12,j}(2a_{N_1-1,j}^n + c\rho_{N_1,j}^n/\Delta\tau) - (r_{12,j}-1)/\Delta x_1) = = r_{12,j}(\gamma_{N_1-1}^{n-\frac{1}{2}} a_{N_1-1,j}^n + \gamma_{N_1,j}^n c\rho_{N_1,j}^{n+1}/\Delta\tau) + \partial S/\partial T_{N_1,j}. \tag{19}$$

For the second time half-step $i = \overline{1,\ N_1}$:

$$\gamma_{i,j}^{n-1} (r_{21}(2b_{i,1}^n + c\rho_{i,1}^n/\Delta\tau) + (r_{21,i}-1)/\Delta x_2) = r_{21,i}(\gamma_{i,2}^{n-1} b_{i,1}^n + \gamma_{i,1}^{n-\frac{1}{2}} c\rho_{i,1}^n/\Delta\tau); \tag{20}$$

$$2\gamma_{i,1}^{n-1} r_{21,i} b_{i,1}^n - \gamma_{i,2}^{n-1} (b_{i,1}^n + b_{i,2}^n + c\rho_{i,2}^n/\Delta\tau) + \gamma_{i,3}^n b_{i,2}^n = -\gamma_{i,2}^{n-\frac{1}{2}} c\rho_{i,2}^n/\Delta\tau; \tag{21}$$

$$\gamma_{i,j-1}^{n-1} b_{i,j-1}^n - \gamma_{i,j}^{n-1} (b_{i,j}^n + b_{i,j-1}^n + c\rho_{i,j}^n/\Delta\tau) + \gamma_{i,j+1}^n b_{i,j}^n = -\gamma_{i,j}^{n-\frac{1}{2}} c\rho_{i,j}^n/\Delta\tau, \quad j = \overline{3,\ N_2-2}; \tag{22}$$

$$\gamma_{i,N_2-2}^{n-1} b_{i,N_2-2}^n - \gamma_{i,N_2-1}^{n-1} (b_{i,N_2-1}^n + b_{i,N_2-2}^n + c\rho_{i,N_2-1}^n/\Delta\tau) + \gamma_{i,N_2}^{n-1} b_{i,N_2-1}^n = -\gamma_{i,N_2-1}^{n-\frac{1}{2}} c\rho_{i,N_2-1}^n/\Delta\tau; \tag{23}$$

$$\gamma_{i,N_2}^{n-1} (r_{22,i}(2b_{i,N_2-1}^n + c\rho_{i,N_2}^n/\Delta\tau) - (r_{22,i}-1)\,\Delta x_1) = r_{22,i}(\gamma_{i,N_2-1}^{n-1} b_{i,N_2-1}^n + \gamma_{i,N_2}^{n-\frac{1}{2}} c\rho_{i,N_2}/\Delta\tau). \tag{24}$$

After each whole time step, the computed values of $\gamma_{i,j}^n$ are used to determine the gradients of the functional (7) relative to the values of thermal conductivity $\lambda_{x_1,i,j}^n$ and $\lambda_{x_2,i,j}^n$ at each node of the difference grid.

Along the axis x_1, $j = \overline{1,\ N_2}$:

$$\partial S/\partial\lambda_{x_1,1,j}^n = c_{1,j}(2\gamma_{1,j}^{n-1} - \gamma_{2,j}^{n-1})(T_{2,j}^n - T_{1,j}^n); \tag{25}$$

$$\partial S/\partial\lambda_{x_1,2,j}^n = d_{1,j}(2\gamma_{1,j}^{n-1} - \gamma_{2-j}^{n-1})(T_{2,j}^n - T_{1,j}^n) + c_{2,j}(\gamma_{3,j}^{n-1} - \gamma_{2,j}^{n-1})(T_{2,j}^n - T_{3,j}^n); \tag{26}$$

$$\partial S/\partial\lambda_{x_1,i,j}^n = d_{i-1,j}(\gamma_{i-1,j}^{n-1} - \gamma_{i,j}^{n-1})(T_{i,j}^n - T_{i-1,j}^n) + + c_{i,j}(\gamma_{i+1,j}^{n-1} - \gamma_{i,j}^{n-1})(T_{i,j}^n - T_{i+1,j}^n), \quad i = \overline{3,\ N_1-2}; \tag{27}$$

$$\partial S/\partial\lambda_{x_1,N_1-1,j}^n = d_{N_1-2,j}(\gamma_{N_1-2,j}^{n-1} - \gamma_{N_1-1,j}^{n-1})(T_{N_1-1,j}^n - T_{N_1-2,j}^n) + + c_{N_1-1,j}(2\gamma_{N_1,j}^{n-1} - \gamma_{N_1-1,j}^{n-1})(T_{N_1-1,j}^n - T_{N_1,j}^n); \tag{28}$$

$$\partial S/\partial\lambda_{x_1,N_1,j}^n = d_{N_1-1,j}(2\gamma_{N_1,j}^{n-1} - \gamma_{N_1-1,j}^{n-1})(T_{N_1-1,j}^n - T_{N_1,j}^n), \tag{29}$$

where

$$c_{i,j} = \frac{2(\lambda_{x_1,i+1,j}^n)^2}{(\lambda_{x_1,i+1,j}^n + \lambda_{x_1,i,j}^n)^2};$$

$$d_{i,j} = \frac{2(\lambda_{x_1,i,j}^n)^2}{(\lambda_{x_1,i+1,j}^n + \lambda_{x_1,i,j}^n)^2}.$$

Along the axis x_2, $i = \overline{1,\ N_1}$:

$$\partial S/\partial\lambda_{x_2,i,1}^n = e_{i,1}(2\gamma_{i,1}^{n-1} - \gamma_{i,2}^{n-1})(T_{i,2}^n - T_{i,1}^n); \tag{30}$$

$$\partial S/\partial\lambda_{x_2,i,2}^n = f_{i,1}(2\gamma_{i,1}^{n-1} - \gamma_{i,2}^{n-1})(T_{i,2}^n - T_{i,1}^n) + e_{i,2}(\gamma_{i,3}^{n-1} - \gamma_{i,2}^{n-1})(T_{i,2}^n - T_{i,3}^n); \tag{31}$$

$$\partial S/\partial\lambda_{x_2,i,j}^n = f_{i-1,j}(\gamma_{i,j-1}^{n-1} - \gamma_{i,j}^{n-1})(T_{i,j}^n - T_{i,j-1}^n) + + e_{i,j}(\gamma_{i,j+1}^{n-1} - \gamma_{i,j}^{n-1})(T_{i,j}^n - T_{i,j+1}^n), \quad j = \overline{3,\ N_2-2}; \tag{32}$$

$$\partial S/\partial \lambda^n_{x_2,i,N_2-1} = f_{i,N_2-2}(\gamma^{n-1}_{i,N_2-2} - \gamma^{n-1}_{i,N_2-1})(T^n_{i,N_2-1} - T^n_{i,N_2-2}) + $$
$$+ e_{i,N_2-1}(2\gamma^{n-1}_{i,N_2} - \gamma^{n-1}_{i,N_2-1})(T^n_{i,N_2-1} - T^n_{i,N_2}); \quad (33)$$

$$\partial S/\partial \lambda^n_{x_2,i,N_2} = f_{i,N_2-1}(2\gamma^{n-1}_{i,N_2} - \gamma^{n-1}_{i,N_2-1})(T^n_{i,N_2-1} - T^n_{i,N_2}), \quad (34)$$

where

$$e_{i,j} = \frac{2(\lambda^n_{x_2,i,j+1})^2}{(\lambda^n_{x_2,i,j+1} + \lambda^n_{x_2,i,j})^2};$$

$$f_{i,j} = \frac{2(\lambda^n_{x_2,i,j})^2}{(\lambda^n_{x_2,i,j+1} + \lambda^n_{x_2,i,j})^2}.$$

We can change over from the calculated values of the gradients for the node i, j to gradients relative to the parameters of the sought relations $\lambda_{\alpha k}$:

$$\partial S/\partial \lambda_{\alpha k} = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sum_{n=1}^{N_3}(\partial S/\partial \lambda^n_{x_\alpha,i,j})(\partial \lambda^n_{x_\alpha,i,j}/\partial \lambda_{\alpha k}), \quad (35)$$
$$\alpha = 1,\ 2; \quad k = \overline{1,\ K},$$

where $\partial S/\partial \lambda_{x_\alpha,i,j}{}^n$, $\alpha = 1, 2$, are calculated by means of Eqs. (25)-(34).

The quantity $\partial \lambda_{x_\alpha}{}^n/\partial \lambda_{\alpha k}$, $\alpha = 1, 2$, is the ratio of the increment of thermal conductivity at the node i, j of the difference grid to the increment of the k-th parameter approximating the temperature dependence of thermal conductivity (8). In the case of a linear change in λ_{x_α} between the nodal values $\lambda_{\alpha k}$, $\alpha = 1, 2$, the quantity $\partial \lambda_{x_\alpha,i,j}/\partial \lambda_{\alpha k}$ is determined from the conditions

$$\frac{\partial \lambda^n_{x_\alpha,i,j}}{\partial \lambda_{\alpha k}} = \begin{cases} 0, & T^n_{i,j} \leqslant T_{k-1}; \\ \dfrac{T^n_{i,j} - T_{k-1}}{T_k - T_{k-1}}, & T_{k-1} < T^n_{i,j} \leqslant T_k; \\ \dfrac{T_{k+1} - T^n_{i,j}}{T_{k+1} - T_k}, & T_k < T^n_{i,j} < T_{k+1}; \\ 0, & T^n_{i,j} > T_{k+1}. \end{cases}$$

The sought parameters $\lambda_{\alpha k}$ are calculated from the formula

$$\lambda^{p+1}_{\alpha k} = \lambda^p_{\alpha k} + \vartheta_{\alpha k} Q, \quad \alpha = 1,\ 2;\ k = \overline{1,\ K}. \quad (36)$$

Let us briefly review the main steps in the solution of the ICP. We use the assigned initial temperature dependence of thermal conductivity from the solution of problem (9)-(14) to calculate the temperature field in the specimen during heating. We simultaneously calculate the functional (7). We use the solution of problem (15)-(27) to calculate the conjugate variable $\gamma_{i,j}{}^n$ in each step at each node of the difference grid. Using the values of $\gamma_{i,j}{}^n$ and Eqs. (25)-(34) and (35), we find the values of the gradients of the functional relative to the sought parameters of the temperature dependence of thermal conductivity. The calculated values of the gradients are then used to determine the search direction by the Davidson–Fletcher–Powell method. Employing cubic interpolation, we find the minimum of the functional in the given direction. At the point of the minimum, we determine the new search direction and perform the next iteration of the minimization process. The computing operation is ended when the functional reaches a previously assigned value.

The correctness of the ICP solution was checked by traditional methods [4]. Taking a specimen made of a material with known thermophysical properties, we calculated the temperature field which develops under prescribed heat-transfer conditions. The temperatures at individual points of the specimen were taken as experimental thermograms, and we used these temperatures and the solution of the ICP to determine the relations $\lambda_{x_1}(T)$ and $\lambda_{x_2}(T)$.

The results of a numerical modeling are presented below. A flat rectangular specimen (Fig. 1) was "heated" by a radiative heat flow concentrated in the central region. The maximum heat flux was $1 \cdot 10^7$ W/m². The thermophysical properties of the model material cor-

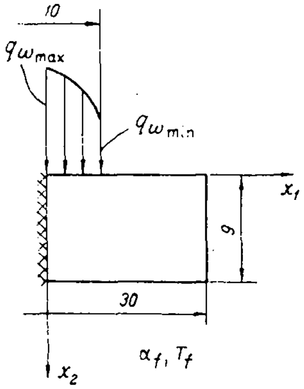

Fig. 1

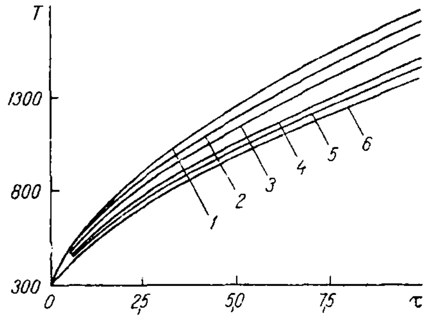

Fig. 2

Fig. 1. Scheme of the numerical experiment: $q_{w_{max}} = 1 \cdot 10^7$ W/m²; $q_{w_{min}} = 0.4 \cdot 10^7$ W/m²; $\alpha_f = 5$ W/(m²·K); $T_f = 293$ K.

Fig. 2. Theoretical thermograms with local heating of the specimen: 1-6) numbers of thermocouples (the coordinates of the thermocouple are shown in Table 1). T, K; τ, sec.

TABLE 1. Coordinates of the Thermocouples

No. of thermo-couple	1	2	3	4	5	6	7	8	9	10
x_1, mm	0	0	0	0	0	0	4,5	4,5	4,5	7,0
x_2, mm	0	0,6	1,5	3,0	3,6	4,8	0	1,5	3,0	3,0

responded to graphite VPP [8]. It was assumed that ten temperature sensors were installed in the specimen (the coordinates of the sensors are shown in Table 1). Heating time τ_e = 10 sec. The temperature measurements were made at 15 time points. The temperature field in the specimen was calculated on a 31 × 31 grid with a time step of 0.5 sec.

In conducting the numerical modeling, calculated values of temperature rounded off to one digit after the decimal point were taken as experimental values. Thus, the maximum error of "measurement" of temperature was 0.05 K. Given the number of temperature measurements (10 points, 15 moments of time), this corresponded to the functional S = 0.4 K². The computation was stopped when the functional reached this value.

We took the same number of parameters K in the approximation of the temperature dependence of thermal conductivity in the temperature range 250-1500 K for the solution of the direct and inverse heat-conduction problems. The chosen number of parameters was six.

To check the uniqueness of the ICP solution, thermal conductivity was determined with two different initial approximations: $\lambda_{x_1} = \lambda_{x_2}$ = 50 W/(m·K) and $\lambda_{x_1} = \lambda_{x_2}$ = 100 W/(m·K). The solution of the ICP required 6 h of machine time on an ES-1030 computer. The time of solution of one direct heat-conduction problem was 4.5 min. The internal memory required was 152 K for the maximum possible number of nodes on a 60 × 60 space grid and 3 MB of disk storage to store the time files. The capacity of the program was 800 operators.

Figures 2 and 3 show the theoretical thermograms corresponding to the numerical experiment. It can be seen that the maximum temperature is 1760 K at the central point of the front surface of the specimen at τ_e = 10 sec. The results of the solution of the ICP (Fig. 4) indicate that the resulting relations $\lambda_{x_1}(T)$ and $\lambda_{x_2}(T)$ correspond very closely to the relations specified in [8] [the deviation within the temperature range 300-1500 K is no more than 0.02 W/(m·K) in each case].

Thus, it has been proven to be possible to comprehensively study heat conduction by orthotropic materials during unsteady high-temperature heating on the basis of the solution of an unsteady two-dimensional coefficient ICP. The above-described algorithm can naturally be extended to axisymmetric bodies. The same principles can be used to construct algorithms

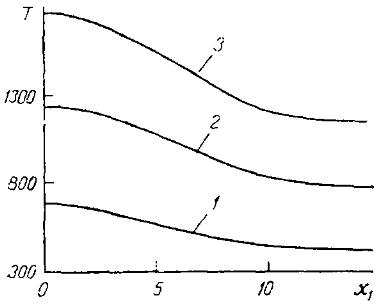

Fig. 3

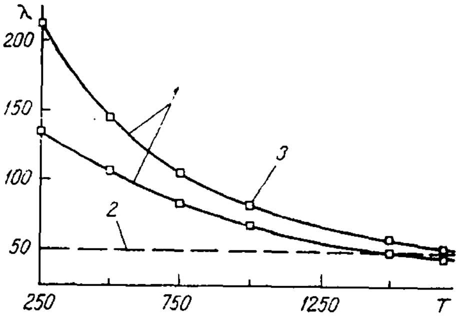

Fig. 4

Fig. 3. Temperature distribution on the front surface of the specimen for different moments of time: 1) $\tau = 1$; 2) 5; 3) 10 sec. x_1, mm.

Fig. 4. Results of solution of the two-dimensional ICP: 1) prescribed relations $\lambda_{x_1}(T)$, $\lambda_{x_2}(T)$, W/(m·K); 2) initial approximation; 3) values obtained from the solution of the ICP.

to determine the thermal conductivity and volumetric heat capacity or emissivity of orthotropic materials. To make reliable use of the algorithm in thermophysical studies, it is necessary to evaluate its boundaries of stability when the initial data contains random and systematic errors.

NOTATION

$\lambda_{x_1}(T)$, $\lambda_{x_2}(T)$, thermal conductivities of the specimen material along the coordinate axes x_1 and x_2, respectively; $c\rho(T)$, volumetric heat capacity of the specimen material; T, temperature; τ, current time; r, coefficient with a value of 0 or 1, depending on the type of boundary conditions; φ, parameter equal to the temperature of the boundary surface or the absorbed heat flux; T_w, temperature of the boundary surface; q_w, incident radiant heat flux; A, ε, absorptivity and emissivity of the given boundary surface; α_f, T_f, heat-transfer coefficient and temperature of the environment; T_e, experimental values of temperature; τ_e, time of experiment; S, functional; M, number of points in specimen at which temperature is measured; $\lambda_{\alpha k}$, sought parameters approximating the temperature dependence of thermal conductivity; K, number of sought parameters; T_k, values of temperature at the nodes of a grid approximating the temperature dependence of thermal conductivity; δT, temperature measurement error; $\bar{\omega}_\ell$, $\bar{\omega}_h$, $\bar{\omega}_\tau$, difference grid; Δx_1, Δx_2, difference steps for the space coordinates; N_1, N_2, number of difference-grid nodes for the coordinates x_1 and x_2; $\Delta\tau$, time step; N_3, number of time steps; ℓ, length of specimen; h, thickness of specimen; $\gamma_{i,j}$, network function of the conjugate equation; p, number of iteration; $\vartheta_{\alpha k}$, element of the vector space; Q, optimum step size in the chosen direction.

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